

EVALUATION OF BOUNDARY CONDITIONS AT THE CLEAR-FLUID AND POROUS-MEDIUM INTERFACE USING THE BOUNDARY ELEMENT METHOD

Hua Tan ¹, Xiao-Min Chen ², Krishna M. Pillai ^{1,3}, Thanasis D. Papathanasiou ²

¹*Department of Mechanical Engineering, University of Wisconsin-Milwaukee
3200 N Cramer St. Milwaukee, Wisconsin 53202 USA*

²*Department of Chemical Engineering, University of South Carolina
301 Main St. Columbia, South Carolina 29208 USA*

³*Corresponding author's Email: krishna@uwm.edu*

SUMMARY: Problems involving fluid flow through a channel bound by porous surfaces are often encountered while modeling flow in liquid composite molding (LCM) processes. In order to conduct flow simulations inside a fiber preform unit-cell, proper boundary conditions need to be applied at the interface between the clear-fluid region and the fibrous porous medium. In this paper, we compare the three well-known interfacial boundary conditions: Beavers-Joseph's slip-velocity condition, Brinkman's stress-continuity condition, and Whitaker's stress-jump conditions. In order to estimate the accuracy of these boundary conditions, a model porous medium made of parallel impermeable cylinders with periodic distribution, is created next to an empty channel for simulation of 2D Stokes flow in the entire pore-space domain. The boundary element method (BEM) is used to solve the Stokes equations in such a domain. Volume averaging allows then recovering the macroscopic average velocities from the pointwise BEM solution. This 'exact' solution is then compared with the analytical solutions obtained for the three considered boundary conditions. Our results show that when the porosity is high, the stress-jump condition leads to the most accurate flow prediction in the interface region. For the low or moderate porosities encountered in the typical LCM applications, none of the three considered conditions can predict the flow behavior at the interfacial region accurately, though they all result in a good estimation of the boundary-layer thickness and of the slip velocity.

KEYWORDS: Liquid Composite Molding (LCM), porous medium, Brinkman equation, Boundary Element Method (BEM), volume averaging

INTRODUCTION

For a steady-state creeping flow through a channel with an adjacent porous medium which is often encountered in liquid composite molding processing, the Stokes equation (Eqn. 1) and Darcy's law (Eqn. 2) can be used to model the clear fluid region and the flow in the isotropic porous medium, respectively:

$$-\nabla p + \mu \nabla^2 \mathbf{u} = 0 \quad (1)$$

$$\langle \mathbf{u} \rangle = -\frac{K}{\mu} \nabla \langle p \rangle^f \quad (2)$$

where \mathbf{u} and $\langle \mathbf{u} \rangle$ respectively represent the point-wise and volume-averaged velocities, p and $\langle p \rangle^f$ respectively represent the point-wise and pore-averaged pressures, K is the isotropic permeability of the porous medium, and μ is the fluid viscosity. Here we face the problem of defining the boundary conditions at the interface between the clear-fluid and porous-medium domains. Beavers and Joseph first proposed the slip velocity boundary condition at the interface [1] as:

$$\left. \frac{du}{dy} \right|_{y=0} = \frac{\alpha}{\sqrt{K}} (u(0) - \langle u \rangle_D) \quad (3)$$

where u is the tangential channel velocity beyond the interface for $y > 0$, $u(0)$ is the tangential channel velocity at the interface, $\langle u \rangle_D$ is the tangential volume-averaged velocity inside the porous medium given by Darcy's law, K is the permeability, and α is the slip coefficient. Saffman [2] mathematically justified the slip condition and showed that the slip coefficient depends on the location of the interfacial boundary. Sahraoui and Kaviany [3] found that the slip coefficient depends on many factors such as porosity, Reynolds number, channel size, and interface location. If the Brinkman equation is used to model the porous-medium flow as:

$$-\nabla \langle p \rangle^f + \mu' \nabla^2 \langle \mathbf{u} \rangle - \frac{\mu}{K} \langle \mathbf{u} \rangle = 0 \quad (4)$$

where μ and μ' represent respectively the fluid and effective viscosities, both the velocity and the shear stress are often assumed to be continuous at the interface. However, estimation of the effective viscosity μ' remains problematic. Neale and Nader proved that if $\alpha = (\mu'/\mu)^{0.5}$, the Brinkman equation with continuity conditions yields the same solution as the Darcy's law with the slip velocity condition [4]. Durlofsky and Brady [5] concluded that Brinkman equation is only valid for dilute system. Sahraoui and Kaviany [3] revealed that the Brinkman equation with a constant effective viscosity predicts correctly the slip velocity but generally does not result in a correct velocity profile in the porous medium near the interface. James and Davis [6] investigated the flow behavior at the interfacial region by analytically solving the Stokes flow in a 2D channel partially made of a periodical array of cylinders. Their numerical results showed that for $\mu' / \mu = 1$, the Brinkman equation over-predicts the velocity at the interface and the boundary penetration depth. Tachie et al. [7] validated experimentally the results of [6] using Particle Image Velocimetry (PIV) in a rotating cylinder apparatus.

Unlike the above mentioned studies, which assumed stress continuity at the interface, Ochoa-Tapia & Whitaker [8] developed a new interfacial boundary condition, using the rigorous volume averaging method while incorporating a stress-jump at the interface. Their results showed that the coefficient β in the stress-jump boundary condition remains of the order of one even for a large variation in the porous medium permeability, whereas the slip coefficient α based on the Beavers-Joseph slip-velocity boundary condition changes over a larger range. Since the slip coefficient α can be related to the effective viscosity through $\alpha = (\mu'/\mu)^{0.5}$ [4], this means that the effective viscosity in the Brinkman equation incorporating the stress-continuity assumption changes over a larger range as well. Therefore, Ochoa-Tapia & Whitaker claimed that their interfacial boundary condition is a more precise representation of the flow physics at the interface.

This brief literature review lists several options to model the flow at the clear-fluid porous-medium interface. But the question remains as to which one of these models is really in best agreement with the actual microscopic flows occurring near such interfaces. The current work is an attempt to answer this question. We use the boundary element method (BEM) to solve the ‘experimental’ 2D Stokes flow equations in a region, part of which is an open channel and part of which is a periodic porous medium made from a square array of fixed parallel cylinders. The volume averaging method is employed using the representative elementary volume (REV) to generate the averaged velocity profile from the point-wise BEM solution. The efficacy of the various interface flow models is evaluated by comparing their predictions with this BEM result. The effect of two different types of REV, the fixed and variable types, on the average velocity profile is discussed as well. Additional details on the material presented in this condensed paper can be obtained from [9].

THEORY

Solution of Brinkman Model Based on the Stress-Continuity Interfacial Condition

The Stokes and Brinkman equations are used to model the steady-state creeping flow in the channel and porous medium, respectively. The interface between the channel and porous medium is located at $y = 0$. The width of the channel is taken to be h . If the velocity and stress are assumed to be continuous at the interface between the clear-fluid (channel) and porous-medium domains for the pressure-driven creeping flow, we can obtain the solution in terms of the velocity profiles in the clear-fluid and porous-medium domains as [4]:

$$u(y) = \langle u \rangle_i \left(1 + \sqrt{\frac{\mu'}{\mu}} \frac{y}{\sqrt{K}} \right) + \frac{K}{2\mu} \frac{dp}{dx} \left(2\sqrt{\frac{\mu'}{\mu}} \frac{y}{\sqrt{K}} + \frac{y^2}{K} \right) \quad \text{for } 0 \leq y \leq h \quad (5)$$

$$\langle u \rangle(y) = \langle u \rangle_i + (\langle u \rangle_D - \langle u \rangle_i) \left[1 - \exp\left(\sqrt{\frac{\mu'}{\mu}} \frac{y}{\sqrt{K}} \right) \right] \quad \text{for } -\infty \leq y \leq 0 \quad (6)$$

where velocity at the interface $\langle u \rangle_i$ is given by:

$$\langle u \rangle_i = \frac{-\frac{K}{2\mu} \frac{d\langle p \rangle^f}{dx} \left(2\sqrt{\frac{\mu'}{\mu}} \frac{h}{\sqrt{K}} + \frac{h^2}{K} \right)}{1 + \sqrt{\frac{\mu'}{\mu}} \frac{h}{\sqrt{K}}} \quad (7)$$

If a boundary-layer thickness δ_c can be defined as the width of the region inside the porous medium near the interface where $\langle u \rangle_i \geq \langle u \rangle \geq 1.01\langle u \rangle_D$, then

$$\delta_c = \sqrt{K} \sqrt{\frac{\mu'}{\mu}} \ln \frac{50 \left(\frac{h^2}{K} - 2 \right)}{1 + \sqrt{\frac{\mu'}{\mu}} \frac{h}{\sqrt{K}}} \quad (8)$$

is obtained from $\langle u \rangle|_{y=-\delta} = 1.01\langle u \rangle_D$ [3]. We can find that the boundary-layer thickness is quite small and is of order $K^{0.5}$. When Darcy's law with the slip boundary condition is used instead of Brinkman equation with the stress-continuity condition, the velocity profile in the clear-fluid region is identical to (5), as long as $\alpha = (\mu'/\mu)^{0.5}$. The difference is that there is no boundary layer inside the porous medium with Darcy's law, while a boundary-layer regime exists inside the porous medium with Brinkman equation. Thus the two approaches can be correlated.

Solution of Brinkman Model Based on the Stress-Jump Interfacial Condition

Ochoa-Tapia and Whitaker [8] proposed a governing equation for flow inside porous medium near the clear-fluid porous-medium interface as:

$$-\nabla\langle p \rangle^f + \frac{\mu}{\varepsilon}\nabla^2\langle \mathbf{u} \rangle - \frac{\mu}{K}\langle \mathbf{u} \rangle = 0 \quad (9)$$

where ε is the porosity of porous medium. When comparing Eqn. (9) and (4), it is obvious that (9) (the *modified Brinkman equation*) is identical to Brinkman equation (the *original Brinkman equation*) as long as $\mu' = \mu/\varepsilon$. Ochoa-Tapia and Whitaker found that the average velocity is continuous at the interface while the stress may not be. Using rigorous volume averaging, they derived a complex mathematical expression to describe a stress jump at the clear-fluid porous medium interface. For 1D parallel flows, the stress-jump interfacial condition reduces to:

$$\frac{1}{\varepsilon} \frac{d\langle u \rangle}{dy} \Big|_{y=0^-} - \frac{du}{dy} \Big|_{y=0^+} = \frac{\beta}{\sqrt{K}} \langle u \rangle \text{ at } y=0 \quad (10)$$

where β is a dimensionless coefficient on the order of one. Using the stress-jump boundary condition (10), the solutions to the Stokes and modified Brinkman equations (1), (9) can be expressed as:

$$u(y) = \langle u \rangle_i \left[1 + \left(\frac{1}{\sqrt{\varepsilon}} - \beta \right) \frac{y}{\sqrt{K}} \right] + \frac{K}{2\mu} \frac{dp}{dx} \left(\frac{2}{\sqrt{\varepsilon}} \frac{y}{\sqrt{K}} + \frac{y^2}{K} \right) \text{ for } 0 \leq y \leq h \quad (11)$$

$$\langle u \rangle(y) = \langle u \rangle_i + (\langle u \rangle_D - \langle u \rangle_i) \left[1 - \exp\left(\sqrt{\varepsilon} \frac{y}{\sqrt{K}} \right) \right] \text{ for } -\infty \leq y \leq 0 \quad (12)$$

where the velocity at the interface $\langle u \rangle_i$ is given by:

$$\langle u \rangle_i = \frac{-\frac{K}{2\mu} \frac{d\langle p \rangle^f}{dx} \left(\frac{2}{\sqrt{\varepsilon}} \frac{h}{\sqrt{K}} + \frac{h^2}{K} \right)}{1 + \frac{h}{\sqrt{K}} \left(\frac{1}{\sqrt{\varepsilon}} - \beta \right)} \quad (13)$$

Using the condition $\langle u \rangle|_{y=-\delta_j} = 1.01\langle u \rangle_D$, the thickness of boundary-layer in the porous-medium flow can be defined as:

$$\delta_j = \sqrt{\frac{K}{\varepsilon}} \ln \frac{50 \left(\frac{h^2}{K} + \frac{2h\beta}{\sqrt{K}} - 2 \right)}{1 + \frac{h}{\sqrt{K}} \left(\frac{1}{\sqrt{\varepsilon}} - \beta \right)} \quad (14)$$

It is clear that if β is zero (i.e, stress is continuous at the interface) and $\mu' = \mu/\varepsilon$, equations (11)-(14) are same as (5)-(8), that is the solution of the modified Brinkman equation using the stress-jump condition reduces to that of the original Brinkman equation with the stress continuity condition. The above solutions to Brinkman equation are in terms of volume averaged quantities, i.e., the solutions are not on the micro (pointwise) scale, but on the macro (REV) scale (see [9] for more details on these analytical results for the stress-continuity and stress-jump conditions).

BOUNDARY ELEMENT METHOD (BEM) FOR STOKES FLOW

In order to study the flow boundary conditions at the clear-fluid / porous medium interface, we place a 2D porous medium, made of a square arrays of cylinders of radii $10 \mu\text{m}$ each, adjacent to a clear-fluid region in form of a channel. The depth h of clear fluid region is $720 \mu\text{m}$. The porosity ε of the porous medium region varies from 0.5 to 0.9 by changing the inter-cylinder space e . The boundary element method (BEM) is employed to solve the pointwise Stokes equation. The code is based on a direct formulation and parallelized with MPI (Message Passing Interface). No-slip boundary conditions are applied at the top and bottom walls $y = h, -W$ and at the surfaces of cylinders. Since pressure-driven flow is considered, a pressure difference Δp is applied on the two vertical boundaries. The pressure gradient $\Delta p/\Delta x$ is 1 Pa/m . The pointwise BEM solution is used as a benchmark to evaluate the Brinkman model with different boundary conditions in this study. More details on this BEM simulation are available elsewhere [9].

VOLUME AVERAGING METHOD

The volume averaging method is employed to average the pointwise BEM solution of the Stokes equation in order to extract the macro-scale flow behavior. Since the flow is 1D on the macro-scale, the rectangular Representative Elementary Volume (REV) used for averaging has the same length as the entire flow domain, while its depth l along the y direction is determined from the required porosity ε . When the REV is moved along the y direction during the averaging, the REV size is kept unchanged (method referred to as the *fixed REV technique*). Sahraoui and Kaviany [3] proposed another type of REV: if the y coordinate of the REV centroid lies between 0 and $-l/2$ near the clear-fluid porous medium interface, the REV dimensions are taken as $-2y$ by L ; otherwise the REV size is still l by L . This type of position-dependent averaging procedure is referred as the *varying REV technique*, to distinguish it from the previous one. Both techniques were used to recover the volume-averaged velocity from the pointwise BEM solution.

RESULTS AND DISCUSSION

In order to determine the effective permeability of a region filled with an array of cylinders, flow in a rectangular domain fully filled with cylinders is separately simulated using the BEM. The permeability of this can be calculated from the average velocity through the domain and the corresponding pressure drop by using the Darcy's law (2). The effective permeability obtained at different porosity levels is listed in Table 1. The numerical results of Sahraoui and Kaviany [3] show that the permeability for a square array of cylinders is given by:

$$K = 0.0602 \frac{\varepsilon^{5.1}}{1-\varepsilon} \frac{\pi}{4} a^2 \text{ for } 0.4 \leq \varepsilon \leq 0.8 \quad (15)$$

where a is the radius of cylinder. Our results show [10] that the permeability estimated by the BEM agrees well with that calculated from (15).

Table 1 Permeability of a square array of cylinders

	$\varepsilon=0.5$	$\varepsilon=0.6$	$\varepsilon=0.7$	$\varepsilon=0.8$	$\varepsilon=0.9$
BEM(m ²)	1.19E-12	3.52E-12	9.9E-12	3.01E-11	1.26E-10
(Eqn. 15) (m ²)	1.10E-12	3.49E-12	1.02E-11	3.03E-11	—

Comparison of the Pointwise Solution with Brinkman model

The effective viscosity μ' must be determined in order to compare the Brinkman solution to the average velocity based on the point-wise solution. Similarly, the adjustable coefficient β in the stress jump condition needs to be determined as well. The parameter μ' is decided by adjusting μ' to match the total flow rate in the channel calculated from (6-7) with that from the BEM solution [9], which is equivalent to the experimental procedure which Beavers and Joseph used to calculate the slip coefficient [1]. The coefficient β in the stress jump condition is determined by the same approach as well. Values of μ' / μ and β thus determined are listed in Table 2. If Darcy's law with the slip-velocity boundary condition is used for the porous medium instead of the Brinkman model with the stress continuity condition, the slip coefficient α is related to μ' by $\alpha = (\mu' / \mu)^{0.5}$. Values of the resulting slip coefficient α are listed in Table 2 as well. The trend of porosity dependence of the slip coefficient from our study agrees well with the one observed by Sahraoui and Kaviani [3], however our slip coefficient values are slightly smaller. This is caused by the inclusion of the inertial terms (Reynolds number $Re \sim 1$) in their simulation, and their results indicate that the slip coefficient increases with the Reynolds number for $0.1 \leq Re \leq 10$.

Table 2 Effective viscosity μ' in (4), coefficient β in (10) and slip coefficient α in (3)

	$\varepsilon=0.5$	$\varepsilon=0.6$	$\varepsilon=0.7$	$\varepsilon=0.8$	$\varepsilon=0.9$
μ' / μ	1.2	1.7	3.2	6.3	14.4
β	0.3	-0.1	-1.0	-1.8	-2.2
α	1.1	1.3	1.8	2.5	3.8

Fig. 1 shows streamlines for the pressure-driven channel flow over square arrays of circular cylinders; only flow details near the top two rows of cylinders are shown. The streamlines above the top row of cylinders are smooth, and the streamline spacing indicates a slight perturbation in the channel flow due to the presence of cylinders below. However below the center-line of the top row of cylinders, the streamlines appear very wavy, indicating a rapidly diminishing x-direction velocity (as spacing increases downward) and a significant transverse velocity around the cylinders. In the second row of cylinders, the effect of the channel flow is almost negligible and the streamline pattern resembles that of the pressure-driven flow through an infinite square array of circular cylinders. In the low porosity ($\varepsilon = 0.5$) case, the outermost recirculation eddies resembles those in the inner row of cylinders, which indicates that the flow field decays very rapidly with a reduction in the porosity.

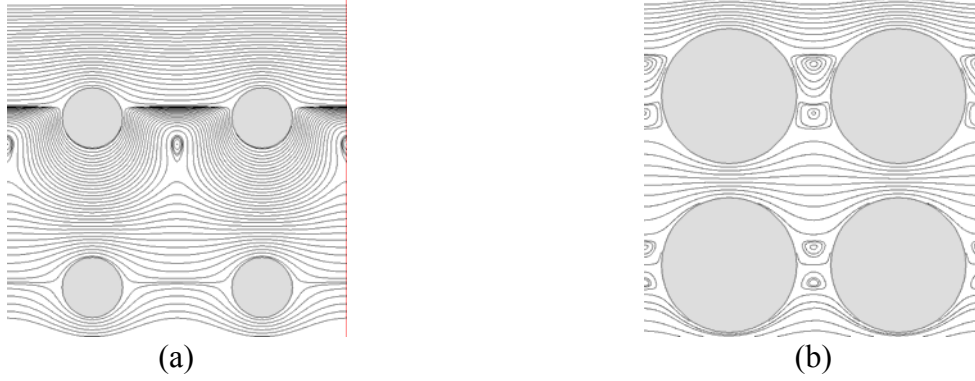


Fig. 1 Streamlines for pressure-driven channel flow (a) $\varepsilon = 0.90$, (b) $\varepsilon = 0.50$.

It has frequently been assumed that the effective viscosity μ' is equal to the fluid viscosity μ . In order to estimate the accuracy of this simplifying approach, we compare its solution with the volume averaged results obtained by the BEM. Fig. 2a, b shows, for $\varepsilon = 0.9$ and 0.5 , the comparison of the volume averaged velocity based on the pointwise BEM solution with the solution of the original Brinkman equation with the stress continuity condition for a suitable μ' (listed in Table 2) or for $\mu' = \mu$, and with the solution of the modified Brinkman equation with stress-jump condition. We find that when porosity ε is 0.9 , solution of the modified Brinkman equation using the stress-jump condition, agrees well with the average velocity profiles based on the pointwise BEM solutions, especially the varying REV result. The slip velocity, the penetration depth of the boundary layer, and the velocity profile are well predicted through the use of the stress-jump condition [9]. With a decrease in the porosity, the modified Brinkman equation with the stress-jump condition progressively fails to predict the volume averaged velocity near the interface; however it still predicts the penetration depth and slip velocity accurately [9]. Fig. 2a, 2b also show that the original Brinkman equation with the stress-continuity equation cannot accurately predict the velocity profiles at the interfacial region for the full range of ε .

Fig. 2 also indicates that the original Brinkman equation with $\mu' = \mu$ and based on the stress continuity condition, severely over-estimates the slip velocity for high porosity porous media (note that the velocity is in log scale). The experiment by Tachie et al. [7] showed that the observed slip-velocity is merely 24-30% of the value predicted by the original Brinkman equation for $\mu' = \mu$ and ε ranging from 0.84 to 0.99 . We also find that the velocity profiles predicted by the Brinkman models with the stress-continuity and stress-jump conditions get closer to each other for low porosity ($\varepsilon < 0.7$) conditions. Thus, we conclude that for a high porosity ($\varepsilon > 0.8$) medium, the modified Brinkman equation with the stress-jump condition predicts velocity at the interfacial region accurately, while the original Brinkman equation with the stress-continuity condition fails to do so. When the porosity is low ($\varepsilon < 0.8$), although the Brinkman models, with the stress-jump or with the stress-continuity boundary conditions, may correctly estimate the penetration depth and slip velocity, none of them can accurately predict the flow velocities at the interfacial region. Therefore, we can conclude that various Brinkman models are not valid for lower porosity ($\varepsilon < 0.8$) media as far as flow prediction in the interfacial region is concerned. But if only the penetration depth and slip velocity are of interest in such media, the original Brinkman equation with $\mu = \mu'$ can yield fairly good estimates.

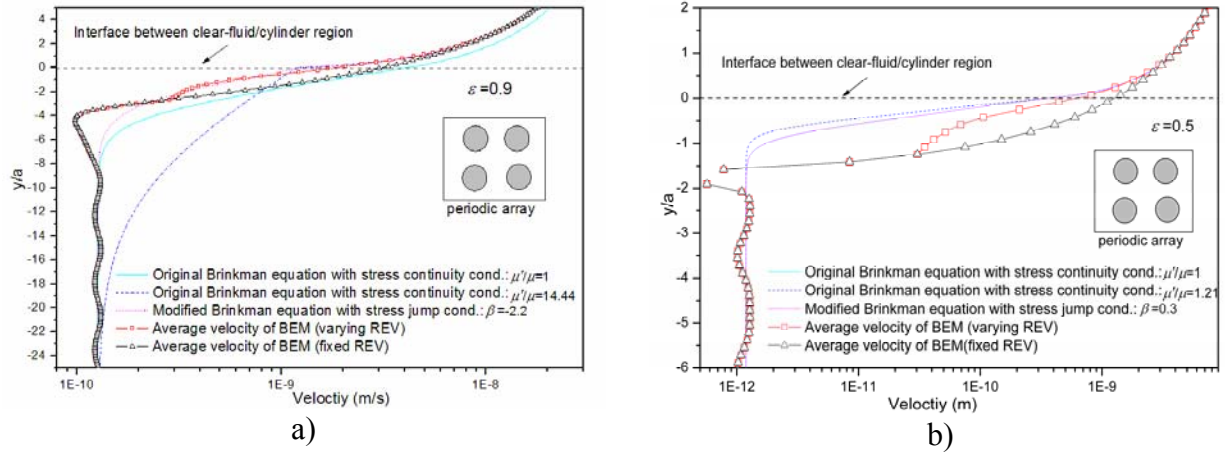


Fig. 2 Comparison of average velocity based on the pointwise BEM solution with the Brinkman models using different interfacial conditions: (a) $\epsilon = 0.9$; (b): $\epsilon = 0.5$.

Average Velocity Profile of Pointwise Solution Affected by Volume Averaging

In Fig. 2, the velocity profiles recovered using the varying and fixed REV techniques show considerable disparity in the interfacial region. We choose REV's with different depths l , $2l$, $3l$ and $4l$ (see [9] for details) in the averaging process to show the influence of REV size on the average velocity. The comparisons of the average velocity profiles for $\epsilon = 0.9$ using the varying REV are plotted in Fig. 3. The averaged BEM results are compared with the modified Brinkman equation predictions as the latter was found earlier to be the most accurate for high porosity media. It is clear from Fig. 3 that the REV size does have an influence on the average velocity profile. Note that when the varying REV technique, the average velocity profiles in Fig. 3 have a very similar feature: the average velocity drops sharply to a low value, which is even lower than the Darcy velocity found farther away from the interface; the average velocity then rises to recover the Darcy value either in a steady monotonic or fluctuating manner. This increasingly fluctuating velocity helps to judge the correct penetration depth, while the immediate drop to the Darcy's velocity witnessed in the larger REV velocity profile provides false information on the flow behavior at the interface. Therefore, too large a REV masks the increasingly fluctuating region and makes it difficult to discern the real penetration depth. The same conclusion can be drawn for the fixed REV technique [9]. Both the varying and fixed REV techniques show that an optimum REV width is required to reflect accurately the flow behavior at the interfacial region; for high porosity, suitable REV depth lies between l to $3l$; for low porosity, the suitable depth is l . Farther away from the interface, the velocity profiles based on different REV sizes are almost identical.

CONCLUSIONS

The boundary conditions at the interface between the clear-fluid / porous-medium regions have been studied in detail. The velocity profiles obtained from the original and modified Brinkman equations (using either the stress-continuity or stress-jump interfacial conditions) are compared with the volume averaged velocity based on the pointwise Stokes velocity obtained numerically by the boundary element method (BEM). Our results show that when the porosity is high, the modified Brinkman equation with the stress-jump boundary condition gives the most accurate flow prediction at the interfacial region. For low or moderate porosities, although the two

Brinkman models with stress-continuity or stress-jump conditions do not predict the flow behavior accurately at the interfacial region, they all yield a good estimate of the boundary-layer thickness and slip velocity. So it is acceptable to use the fluid viscosity to obtain accurate boundary-layer thickness and interfacial velocity. The varying REV technique generates also a velocity profile at the interfacial region different from the one obtained by the fixed REV technique. The REV size has a significant influence on the average velocity profile: wider REV sizes lead to excess smoothing that in turn mask the interfacial effects on the velocity profile.

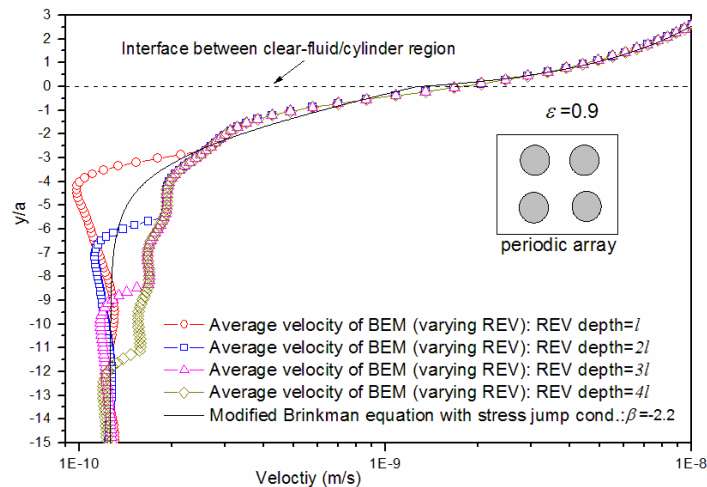


Fig. 3 Comparison of velocity profiles averaged using different REV sizes.

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